

DEVELOPING MATHEMATICAL REASONING USING ATTRIBUTE GAMES

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The game of SET[®] has proven to be a very popular game at our college mathematics club meetings. Since we've started playing, the membership has grown every month. In fact, one of our members brought her six year old son to a meeting, and he now looks forward to playing SET[®] with us every month. As a result of playing the game in our club and thinking about the results, we created and solved a variety of mathematical questions. For example, we wondered about possible strategies for winning and conjectured about phenomena that happened when playing. These questions involve a wide variety of traditional mathematical topics, such as the multiplication principle, combinations and permutations, divisibility, modular arithmetic, and mathematical proof.

In addition to encouraging the posing and solving of these problems in our math club, we took the game and our questions into our classrooms to see what reasoning could be encouraged. We tested our original questions on several groups of junior high and high school students and on several hundred freshmen and sophomore college students who were not mathematics majors.

The purpose of this article is to show how games such as SET[®] can be used to develop mathematical reasoning by describing student investigative work that has resulted from playing the game. After giving a description of the game, we will pose and answer some of the questions that were solved by members of our club and by students ranging in academic level from ninth grade to college. We will also describe what teachers can do to facilitate the development of reasoning using this game. Although this article discusses problems that were generated from the game of SET[®], any game that uses attributes can be used to stimulate logical thinking.

Introduction to the game of SET[®]: Twelve cards are placed face up on the table. Each player looks for a "SET" of three cards. When players find *SETs*, they put them in their own piles, and three new cards are put on the table to bring the total back to twelve. The game continues until all of the cards are dealt and no more *SETs* are found. If at any time there are no *SETs*, three more cards are added until a *SET* can be found. The player with the most *SETs* at the end of the game wins.

The deck of cards: Each card can be identified by four attributes, each of which has three values: number (1, 2, 3), color (red, green, purple), symbol (diamond, oval, squiggle), and shading (open, striped, solid). The deck is made up of one card of each type. For example, there is one card that has 1 red diamond with solid shading. (We will call it: one-red-diamond-solid.)

Sets: Three cards make a set if, for each attribute, the values on the cards are either all the same or all different. To illustrate, consider the following example.



a



b



c



d

Figure 1: Examples and non-examples of *SETs*

The three cards in Figure 1a are a *SET* because their symbols are all the same (diamonds), their shadings are all the same (solid), their numbers are all different (1, 2, 3), and their colors are all different (red, purple, green). The three cards in Figure 1b are a *SET* because their numbers are all different, their colors are all different, their symbols are all different, and their shadings are all different. However, the three cards in Figure 1c are not a *SET* because their symbols are neither all the same nor all different (2 diamonds and 1 oval). Similarly, the three cards in Figure 1d are not a *SET* because the colors are neither all the same nor all different (2 red and 1 green). Thus, for any attribute, if the values are the same for exactly 2 of the 3 cards, the 3 cards are not a *SET*.

A summary of the questions we posed is seen in Table 1.

	QUESTION
#1	Find as many <i>SETs</i> as possible in Figure 2.
#2	How many cards must be in the deck?
#3	How many <i>SETs</i> (including overlapping ones) are possible in the deck?
#4	What is the best strategy when searching for <i>SETs</i> ? Which type are you most likely to find?
#5	What is the average number of <i>SETs</i> among 12 randomly selected cards?
#6	If one attribute is fixed, how many cards could there be that contain no <i>SETs</i> ?
#7	Find as many cards as possible that contain no <i>SETs</i> .
#8	Can only three cards be left at the end of the game?

Table 1:

Questions posed to our junior high, high school, and beginning college students

Question #1: Find as many *SETs* as possible in Figure 2.

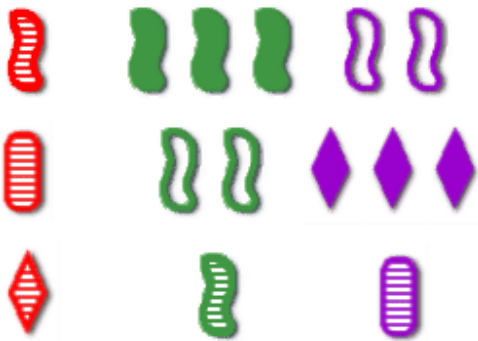


Figure 2: Find as many *SETs* as you can

Answer #1: There are six *SETs*, one in every row and one in every column.

Student Work on #1: While searching for *SETs*, students developed several strategies that also helped them answer the later questions. Sara, a ninth grade algebra student, started by looking at two cards and then determining what the third card must be to form a *SET*. Corey looked only for *SETs* that had cards that differed in number. Kathleen scanned the entire group of cards for similarities. For example, she looked for *SETs* among the solid cards.

While playing once, they saw that there were no *SETs* in the last 12 cards, as seen in Figure 3. The students built on the above strategies in order to try to prove this. Corey said that there was only one card with 1 figure left and that it did not form a set with any of the other cards. Kathleen said that there was only one striped card. Since it didn't form a *SET* with any of the other cards, then any possible *SET* must be made of three solid cards or three open cards. Similarly, since the one squiggle card didn't form a *SET* with any of the other cards, possible *SETs* must be all diamonds or all ovals.

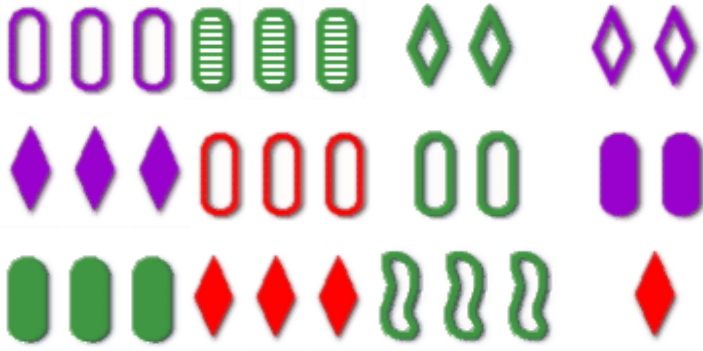


Figure 3 : Prove that there are no *SETs* among these cards

Some students had difficulty playing or answering questions if they considered all four attributes. Initially, this frustration can be reduced by considering only the red cards.

Question #2: How many cards must be in a deck?

Answer #2: Since each of the four attributes has three values, there must be $3^4=81$ cards.

Student Work on #2: Students at all levels had no problem with this question. Most of the ninth grade students solved this problem like Sara did. She started with one card: one-red-oval-open. She then considered the cards that differed from this one in shape and then in number. She drew these 9 cards, as in Figure 4. Then she said these could also be striped or solid, hence $9 \times 3=27$, and that the 27 cards could also be green or purple, hence $27 \times 3=81$.



Figure 4: There are 9 open-red cards

Most of the college freshmen started solving the problem by constructing a tree diagram, where attributes were considered one at a time. The result was $3 \times 3 \times 3 \times 3=81$ cards.

Question #3: How many *SETs* (including overlapping ones) are possible in the deck?

Answer #3: For each of the 81 cards, any of the remaining 80 could be used to make a *SET*. Once these two are chosen, only one card exists to complete the *SET*. In fact, a good exercise to improve one's speed is to take two cards and name the third card which completes the *SET*. To illustrate, if the two cards are the ones seen in Figure 5a, the numbers are the same (two), the colors are different (red,

purple), the symbols are different (squiggle, diamond), and the shadings are different (striped, solid). So, the third card is the one in Figure 5b (two-green-oval-open).



a



b

Figure 5: Two cards in a *SET* determine the third

So there are $81 \times 80 \times 1 = 6480$ possibilities. However, since the order of the cards in the *SET* doesn't matter, there are $6480/3! = 1080$ possible sets.

Student Work on #3: Sara knew that once the first two cards had been selected that the third card of the *SET* was determined. She started this problem by focusing on one card: one-open-green-diamond. She noted that the second card could be one of the following cards: two-open-green-diamond (varying the number), one-open-red-diamond (varying the color), one-open-red-oval (varying the symbol), or one-striped-red-diamond (varying the shading). She concluded in a matter of minutes that the first card could be chosen 81 ways, the second card 4 ways, and the third card one way. So she answered $81 \times 4 = 324$. Corey and Kathleen noticed that the second card could be chosen in more than 4 ways, by changing more than one attribute at a time. They spent several minutes coming up with as many second cards as they could. After becoming frustrated with drawing out all of their second cards, Kathleen realized it would be any of the remaining 80 cards. Corey concluded there would be $81 \times 80 = 6480$ *SETs*. The teacher asked if they counted any of the *SETs* more than once. They looked at one sample *SET* and saw that by moving the cards around that it would have been counted 6 times. So Corey concluded $81 \times 80 / 6 = 1080$.

Some of the college students took considerably more time than the ninth graders to correctly count the total number of *SETs*. Although early on Kathryn and Angela, college sophomores, realized that the second card could be any of 80, they became very concerned about counting the same set more than once. In their attempt to avoid overcounting, they systematically arranged all of the 81 cards into 3 groups, according to symbol. By doing this, they were actually building ideas that would help them easily answer later questions, such as question #4. After correctly counting all of the *SETs* that could contain one fixed card, one by one, they got frustrated about whether they would overcount while generalizing to a problem that contained all 3 symbols. After the teacher suggested that they worry about overcounting at the end of the problem, they easily came up with the correct answer. Kathryn even made a connection between the 6 ways one *SET* could be written and the way she represented electron configurations in her chemistry class.

This was also a very good question to ask the college freshmen who had studied probability and statistics in class. Many initially suggested that the answer might be $81C3$ ($81! / [(81-3)!3!] = 85,320$), the number of possible groups of three cards. When asked whether these were all *SETs*, they realized

that each group of 3 did not necessarily make a *SET*. Some students then said the answer must be $81P_3$ (or, equivalently $81 \times 80 \times 79 = 511,920$), since these were the two techniques they knew best. After discussion, they realized that this was not a straightforward problem. Playing the game was instrumental in solving the problem. Those that searched for *SETs* by starting with one card at a time (instead of scanning for general patterns) were most successful in arriving at: $81 \times 80 \times 1$. Some needed to be asked whether they had overcounted before they thought to divide by 6. Working in groups, most of the class had success on this problem, and many commented that the problem really made them think.

Question #4: What is the best strategy when searching for *SETs*? Which type are you most likely to find?

Answer #4: On each of the four attributes, the values must be all the same or all different. So, *SETs* are of the following types: 4 different, 3 different and 1 same, 2 different and 2 same, or 1 different and 3 same. We'll determine how many of each type exist with the full deck of 81 cards. For the "4 different" type, consider that the first card can be any of the 81. The second card must be different on all attributes. So there are $24 = 16$ possible cards (2 colors, 2 shadings, 2 symbols, 2 numbers). The third card is determined. As with the total number of cards, each *SET* has been counted $3! = 6$ times. So there are $(81 \times 16 \times 1) / 6 = 216$ *SETs* of this type.

For the "3 different and 1 same" type, the first card can be any of the 81. The second card will stay the same on one of 4 attributes. If, for example, the common attribute is color, the second card can be any of $23 = 8$ (2 shadings, 2 symbols, 2 numbers). So the answer is $(81 \times 4 \times 8) / 6 = 432$. For the "2 different and 2 same" type, again the first card can be any of 81. There are $4C_2 = 6$ ways to select the two attributes that remain the same for the second card. If, for example, these are color and symbol, there are $22 = 4$ ways to select the second card (2 shadings, 2 numbers). So the answer is $(81 \times 6 \times 4) / 6 = 324$. The last type could be determined in a similar manner. The likelihoods change during the course of the game as *SETs* are removed. However, the likelihoods that hold at the beginning of the game are summarized in Table 2.

TYPE OF SET	WAYS TO PICK 1st CARD	WAYS TO PICK THE ATTRIBUTE THAT IS SAME ON THE 2nd CARD	WAYS TO PICK THE 2nd CARD	WAYS TO PICK THE 3rd CARD	NUMBER OF SETS OF THIS TYPE	LIKELIHOOD OF THIS TYPE OF SET
4 different/ 0 same	81	$4C_0 = 1$	$2^4 = 16$	1	$(81 \times 16) / 6 = 216$	$216 / 1080 = 20\%$
3 different/ 1 same	81	$4C_1 = 4$	$2^3 = 8$	1	$(81 \times 4 \times 8) / 6 = 432$	$432 / 1080 = 40\%$
2 different/ 2 same	81	$4C_2 = 6$	$2^2 = 4$	1	$(81 \times 6 \times 4) / 6 = 324$	$324 / 1080 = 30\%$
1 different/ 3 same	81	$4C_3 = 4$	$2^1 = 2$	1	$(81 \times 4 \times 2) / 6 = 108$	$108 / 1080 = 10\%$
TOTAL					1080	

Table 2: Probabilities of different "types" of *SETs*

Student Work on #4: Students were very interested in posing and solving this problem since each of the students had commented while playing the game that certain "types" of *SETs* seemed more common. Working in a group, members of the math club solved this problem as shown above in about 35 minutes. They tried a few cases with the cards before they were able to come up with these general answers. They saved the hardest case (2 different and 2 same) for last and had already deduced the answer should be $1080 - (216 + 432 + 108) = 324$. Jaclyn and Paul were very excited when they were able to prove why this answer was 324, as seen in Table 2. While solving the problem, Mike and Cathy posed questions such as: "Should the answer for 3 different be the same as 3 same?", and "Should 4 different be the same as 4 same?" (until they realized that "4 same" could not occur since there was only one of each type of card).

One group of ninth grade students (Sara, Kathleen, and Corey) came up with an answer similar to the one above in about 1 hour. The teacher suggested starting with the easier cases. For "4 different", they started with one card: one-open-red-diamond. Then they laid the cards out on the table and made every possible *SET* of this type with this first card, realizing that they couldn't choose a second card which had the same value on any of the 4 attributes. So the second card had to be green or purple, squiggle or oval, solid or striped. Instead of dividing by 6 at the end, the ninth graders didn't overcount. They let the second card have 2 figures, and then the third card was determined. So they came up with the $2 \times 3 = 8$ second cards systematically and concluded that the first card (having 1 figure) could be any of 27. So the answer was $27 \times 8 = 216$. They got each of the above answers, sometimes needing to be reminded that the different attributes could be any of 4, instead of just the one they were discussing. They left "2 different and 2 same" for last, and simply answered that it was $1080 - (216 + 432 + 108) = 324$, as the math club had also conjectured. Then they proceeded to come up with the correct probabilities for each type. Note that none of these students had ever studied combinatorics and that there were only minor suggestions made by the teacher. The student's success in creating these general arguments seemed to come from the fact that they could use the cards to first create special cases and from the insight they had gained from noticing and looking for different types of *SETs* while playing the game.

Question #5: What is the average number of *SETs* among 12 randomly selected cards?

Answer #5: Given any two cards, there are 79 cards remaining and exactly one of these completes the *SET*. It follows that the probability of any three cards making a *SET* is $1/79$. Since the number of possible combinations of three cards chosen from twelve cards is $12C3$ ($12C3 = 12! / [(12-3)!3!] = 220$), the expected number of *SETs* in a group of 12 cards is $1/79$ times $12C3$, or approximately 2.78. However, note that since the expected number of sets includes overlapping sets that share cards, they cannot all be used when playing the game. In addition, the above argument assumes a full deck of 81 cards. Part way through a game, the results would change.

Student Work on #5: As a result of experience from playing, all groups guessed that the answer would be between 2 and 3. This correct conjecture that was gained from experience helped guide them towards a correct theoretical answer. In the math club, Cathy noticed that the probability of any 3 cards making a set was $1/79$. Since 12 was 4 groups of 3, she concluded that the answer would be $4/79$. They worked together on this (noting the answer should be between 2 and 3), and then Mike realized that there would be $12C3$ possible groups of 3 cards (including overlapping ones). So he concluded $12C3$ times $1/79$, or approximately 2.78.

Vicki, a student in a freshmen statistics course who was familiar with the standard expected value formula [$E(X) = \sum X * P(X)$], had trouble figuring out that X would be 1 each time. The teacher suggested this and then asked how many different groups of 3 existed in the group of 12. This helped

Vicki finish the problem correctly. Jaclyn and Rich, from the math club, also had to discuss what values X and $P(X)$ would have. For those who knew this formula, the question was a good application of the formula. However, some in the math club, such as Mike, came up with the answer without knowing this formula.

Sara, the ninth grade student who had never studied probability, had realized while playing that the first two cards determined the third card. When considering any 3 cards, she quickly said that the chance the third card would complete the set was 1 in 79, but didn't complete the problem.

Question #6: If one attribute is fixed, how many cards could there be that contain no sets?

Answer #6: We proved that 10 cards that share one attribute value must contain a *SET*. However, this is somewhat beyond the scope of this article. So students were just challenged to find as many red cards (fixing the color attribute) that they could that contained no *SETs*. The upcoming student work shows that it is possible to find 9 cards.

Student Work on #6: The ninth graders started with a few red cards that contained no *SETs*. Then they took each of the other red cards from the deck, one at a time, and either tossed it out (if it produced a *SET*) or included it (if it didn't). They came up with 9, but realized they had made one mistake, so they had 8. They wondered if they could exchange the ninth card for one of the cards they had discarded. After several attempts with 9 cards, Corey noted that whenever they had 2 striped, 3 open, and 4 solid cards that they always found a *SET*. He made this conjecture after trying many different cases, and then he provided a partial proof as he demonstrated more cases. He then generalized on this and said that having 2 oval, 3 squiggle, and 4 diamond cards would be the same mathematically and would cause the same problem. (Corey's realization was actually a crucial conjecture that we used when proving the complicated theorem that 10 cards that had one common attribute value must contain a *SET*. Although a full discussion of the theorem is not included, it is interesting to note that thinking about this game inspired advanced reasoning in junior high and high school students.)

As a group, the ninth graders then tried to get 9 cards by arranging their cards according to attribute. For example, they considered arrays of 9 cards with: 3 of each of the 3 symbols, 3 of each of the 3 shadings, and 3 of each of the 3 numbers. However, these 9 cards always contained a *SET*. After they became frustrated, the teacher suggested that to avoid having 2 striped, 3 open, and 4 solid, they could consider 1 striped, 4 open, and 4 solid. After a few attempts, they came up with the following 9 cards that contained no *SETs*, as seen in Figure 6. Each attribute value occurred either 3, 3, and 3 times or 1, 4, and 4 times.



Figure 6 : Nine red cards which contain no *SETs*

Question #7: Find as many cards as possible that contain no *SETs*.

Answer #7: Miller (1997) and Set Enterprises, Inc. (1998) both illustrate that one can find 20 cards that contain no *SETs*. Miller uses 6 red, 8 green, and 6 purple cards. Both mention that they are currently working on a proof to show that 20 is a global maximum, but neither reports one.² Both also discuss creating a computer program to answer this problem.

Student Work on #7: Students realized from experience that sometimes even 18 cards contained no *SETs*. However, since constructing the group of cards oneself can prove to be a challenge, we just asked students to find as large a number as they could.

Since the ninth graders (Sara, Corey, and Kathleen) had spent a lot of time with just the red cards, Sara quickly generalized on her answer for #6 and answered $9 \times 3 = 27$, since green and purple cards could also be included. However, Kathleen then noticed that they would then have three cards (1 red, 1 green, and 1 purple) that were one-open-oval, thus forming a *SET*. So Sara quickly responded that the 18 red and green cards would contain no *SETs*, as in Figure 7.

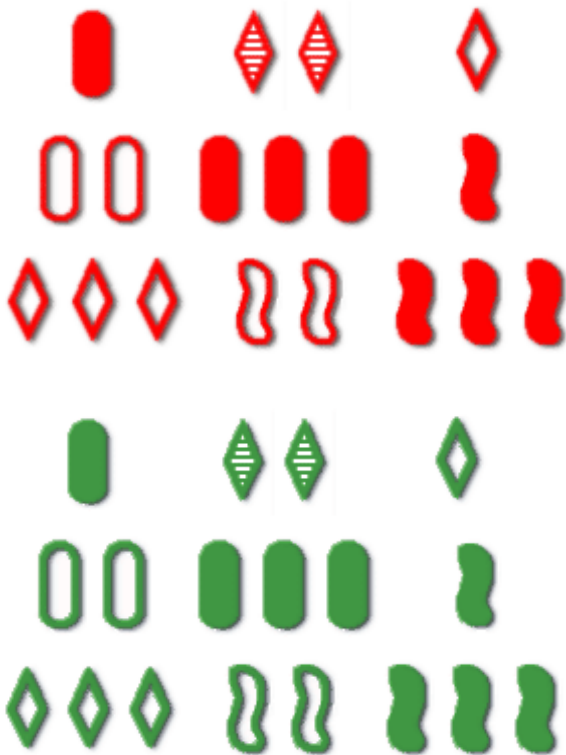


Figure 7: Eighteen cards which contain no *SETs*.

Kathryn and Angela, college sophomores, began this problem with cards of all three colors. They dealt 12 cards and then subtracted any of the cards that would force a *SET* to be found. Then they proved that the 9 cards they had left contained no sets by placing the existing cards into columns according to shading: solid, striped, and open. They noted that any *SET* would have to be contained in a column or would have one card from each column. They quickly went through the rest of the deck, including each card if it did not make a *SET* with any of the cards already on the table. They came up with 18

cards that contained no *SETs*, as seen in Figure 8. Their answer of 18 was a local maximum (since none of the discarded cards could be included), and we discussed how this concept related to calculus. Then Kathryn asked whether there would be another 18 cards that would contain no *SETs*. Immediately she answered her own question by observing that she could change all the solid cards to striped, the striped to open, and the open to solid. Then she and Angela went on to count other similar ways these 18 could be altered. Kathryn was so inspired by thinking about the mathematics involved in *SET*[®] that she searched the Internet for unsolved *SET*[®] problems. She is currently excited about trying to prove one of these unsolved problems and is also thinking about pursuing a mathematics minor.



Figure 8: Eighteen cards which contain no *SETs*

Question #8: Can only three cards be left at the end of the game?

Answer #8: Although the game frequently ends with six or nine cards, we noticed that we never ended with three cards on the table and were inspired to investigate whether this would be possible. The answer proved to be no. To see this, consider the attribute of number. In total, there are 162 figures on the cards. (There are 27 cards with 1 figure, 27 cards with 2 figures, and 27 cards with 3 figures.) Three cards belong to a set with respect to number if and only if the sum of their figures is a multiple of 3. This is because the only possible *SETs* are three cards with 1 figure, three cards with 2 figures, three cards with 3 figures, or one with 1 figure and one with 2 figures and one with 3 figures. Therefore, if the previous 26 *SETs* are valid, the number of figures left for the last three cards will be $162 - 3k$, which is a multiple of 3. Hence, the last three cards do form a *SET* with respect to number. Since the other attributes could be considered in a similar manner, the game cannot end in just 3 cards.

For another approach, consider the attribute of shading. At the beginning of the game, there are 27 solid, 27 striped, and 27 open cards. Notice that $27 \equiv 0 \pmod{3}$. (27 is equivalent to 0 modulo 3. This

means 27 and 0 have the same remainder when divided by 3.) When a *SET* is taken from the deck, we will be left with either 24 of one shading and 27 of the other two shadings ($27 \equiv 24 \pmod{3}$) or with 26 of each shading ($26 \equiv 26 \pmod{3}$). After each *SET* is taken, the numbers of each shading continue to be equivalent modulo 3. So, to end with 2 of one shading and 1 of another would be impossible, because $2 \not\equiv 1 \pmod{3}$. So the last 3 will be a *SET* with respect to shading. Analyzing the other attributes similarly, the last 3 cards will be a *SET*.

Student Work on #8: To simplify the problem, the ninth graders played the game with only red cards. After seeing that the last 3 cards made a *SET*, they noted that among the last three cards, there were 3 cards with 3 figures, 3 cards with diamonds, and 1 card of each type of shading. They said that for the last three not to be a *SET*, there might be, for example, 2 solid and 1 striped. Corey noted that the cards started with 9 of each type of shading. They speculated about possible games that could happen with red cards and then with all cards. The number of each shading as each *SET* is taken are seen in Table 3.

SOLID	STRIPED	OPEN	SOLID	STRIPED	OPEN
9	9	9	27	27	27
6	9	9	26	26	26
6	6	9	23	26	26
3	6	9	23	23	26
3	3	9	23	23	23
3	3	3	20	23	23
2	2	2	20	20	23
1	1	1	20	20	20
0	0	0	17	20	20
			16	16	16
			15	15	15
SOLID	STRIPED	OPEN	15	15	15
9	9	9	12	15	15
8	8	8	12	12	15
8	8	5	12	12	12
5	5	5	9	12	12
4	4	4	9	9	12
3	3	3	9	9	9
0	3	3	8	8	8
0	0	3	5	8	8
0	0	0	5	5	8
			5	5	5
			2	5	5
			2	2	5
			2	2	2
			1	1	1
			0	0	0

Table 3:
The ninth graders show why there cannot be 3 cards left at the end of the game

They discussed the patterns they saw, noting that either 3 was subtracted from one category or 1 from each category. Corey noted that the 27s at the beginning of the game were each divisible by 3. After some discussion, they realized that while the other numbers in the table were not always divisible by 3 that the numbers in each row always had the same remainder when divided by 3. Additional questions motivated by the game, some created by our students, can be seen in Table 4.

ADDITIONAL QUESTIONS
1. Prove that 5 cards that have two common attribute values must include a <i>SET</i> . (For example, consider only the 9 red-open cards, and prove that every group of 5 cards must contain a <i>SET</i> .)
2. Prove that 10 cards that have one common attribute value must include a <i>SET</i> . (For example, prove that 10 red cards must contain a <i>SET</i> .)
3. If 12 randomly selected cards don't contain a <i>SET</i> and 3 additional cards are added, what is the probability of a <i>SET</i> being present?
4. What is the probability that the game will end with 0, 3, 6, 9, & 12 cards?
5. What is the probability of having 2 disjoint <i>SETs</i> among 12 randomly selected cards?
6. Find the maximum number of cards that contain no <i>SETs</i> . Prove that you have a maximum.
7. How does the game change, and how do the answers to some of these questions change if you combine 2 or 3 decks of cards together?
8. Prove that among 7 cards there cannot be exactly 4 <i>SETs</i> .

Table 4:

More challenging questions

Summary of Pedagogical Concerns: The questions discussed were motivated by thinking about the mathematical results of playing the game of SET[®]. We have seen how junior high, high school, and college students have also asked some of these same questions and some of their own after playing the game. Students had no trouble with questions #1-3. Working in groups with the use of actual cards and with the occasional prompting from the teacher, students with no background in combinatorics were able to make progress on each of the questions. Those currently in a probability class were exposed to problems that required more creative thinking than many problems in their textbook. Students that developed certain strategies while playing were successful in proving why sometimes no *SETs* existed or explaining why certain types of *SETs* were more common than other types.

Many of the students made connections among the answers to our questions and connections between these questions and ideas from other math or science courses (such as geometry, calculus, and chemistry). At times the college students took longer to solve the problems than the younger students. However, this was because they carefully considered overcounting issues, and they came up with some of our later questions while trying to answer the earlier ones.

Conclusion: The game of SET[®] provides a multitude of mathematical questions for students of all levels, and students learned a lot of combinatoric theory while having fun. Students developed mathematically sound strategies in order to improve their game. The cards served as manipulatives that students used in order to develop more abstract thinking. Our students enjoyed playing this game, thinking about these questions, and asking their own questions. The game provided an excellent context in which to promote problem solving and deductive reasoning in discrete mathematics, ideas that need to be emphasized in the high school curriculum (NCTM, 1989).

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Endnotes

- 1 The SET[®] game idea and graphics are copyrighted property of SET Enterprises, Inc. and are used for educational purposes by the authors with the written permission (April 13, 1997) of Bob Falco of SET Enterprises, Inc. The SET[®] name and logo are registered trademarks of SET Enterprises, Inc.
- 2 Among interested players on the Internet, the maximum number of cards that contain no set is widely believed to be 20. We searched through relevant journals and on the Internet for a proof. If it has already been proven, this fact has not been widely reported.